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Cylindrical bending of elastic plates

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Abstract

This paper deals with the cylindrical bending of elastic and composite plates subjected to the mechanical transverse loading response under plain strain condition, a complete analytical solution is presented for the cylindrical banding of multilayered orthotropic plates with simply supported edge conditions based on Reissner-Mindlin's first order shear deformation theory (FOST). Composite material is orthotropic in nature and exhibits certain advantages of higher strength and stiffness to weight ratios, longer fatigue life, enhanced corrosion resistance etc. Laminated plate consists of homogeneous elastic laminae of arbitrary thickness. Composite laminates are widely used in construction of mechanical, aerospace, marine and automotive structures. In this formulation, a two dimensional (2D) elasticity problem reduces to one dimensional (1D) plate problem. Excel programming is used to execute the problem. Results of displacements and stresses are presented for simply supported isotropic and orthotropic plates and compared with exact and other available solutions from the literature.

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Keywords: Cylindrical bending; Laminated Composites; Plain strain; Reissner–Mindlin platetheory

1. Introduction:

A flat plate is a structural member having thickness is less than other two dimensions (length and width). Flat plates are extensively used in many engineering applications like tank bottom, floors and roof of the building, deck slabs of the bridges, turbine disks etc. Plates used for these applications are subjected to lateral loads that causing bending deformation as well as stretching. The geometry of the plate is normally defined by the middle plane which is equidistant from top and bottom of the edges of the plate. Thickness of the plate is always measured in the direction of the middle plane. The flexural strength is totally depends on the thickness of the plate. In recent years, the utilization of advanced composite materials is being used increasingly in many of structural applications such as high performance structures. A composite material is obtained by combining two or more materials so that the properties of composites are different from individual constituent material, Due to the special properties exhibited by these new materials, the conventional methods of analysis become inadequate. Very often the structures are subjected to both static and dynamic loads of various magnitudes and complexities. To investigate the actual behaviour of the structures under these loads, rigorous analysis is required to assess the strength and stability under various boundary conditions

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and loading cases. An effective and efficient use in structural applications requires good understanding of their static and dynamic behaviour under various types of loading conditions. It is a challenging problem to understand the dynamical behaviour of composite or sandwich plates with sufficient accuracy.

Laminated composite structures are widely used in many of engineering applications such as civil, mechanical, nuclear, aerospace, chemical industries as well as in sports and health instrument applications due to low specific density and low specific modulus. Laminated composites and sandwich structures constitute light weight with high stiffness, high structural efficiency and durability. Advanced composite materials like graphite/epoxy, boron/epoxy, Kevlar/epoxy etc. are replacing metals and alloys in the manufacturing of structural members. High ratio of inplane modulus to shear modulus of composite laminates, the shear deformation effects are obvious in the thick composite laminates and hence any analytical model should predict accurate of interlaminar stresses in the laminate. These composite materials permit the designer to 'tailor make' the structural properties through various lamination schemes to achieve the specified objectives.

Isotropy property implies that the material properties at a point are the same in all directions. However, some materials have properties that are not independent of direction and such materials are called anisotropic materials. When the material properties are different in two mutually perpendicular directions is called orthotropic materials. There are two types of orthotropy, namely material orthotropy and structural orthotropy. Material orthotropy is due to physical structure of the material e.g. wood, crystals etc., while structural orthotropy is due to fabrication methods used for making structural component e.g. reinforced concrete slabs, fiber reinforced plastics, stiffened plates etc. Ghugal and Shimpi (2002) presented A review of displacement and stress based refined theories for isotropic laminated plates is presented. A refined hybrid plate theory for composite laminate with piezoelectric laminae studied by Mitchell and Reddy (1995).Reissner and Mindlin are the firstly given FOSDT , based on the assumed stress and displacement fields by Chandrashekhar (2001).Bending analysis of a moderately thick orthotropic sector plate subjected to various loading Conditions with the help of first order shear deformation theory studied by Aghdam M and Mohammadi M (2009).Bhar et al.(2009) Significance of using higher-order shear deformation theory (HOSDT) over the first order shear deformation theory (FOSDT) for analyzing laminated composite stiffened plates is brought out using the finite element method (FEM).Vel et al.(2004) gives an Analytical solution for the cylindrical bending vibration of piezoelectric composite plates. The generalized plane deformations of linear piezoelectric laminated plates and cylindrical bending of laminated plates with piezoelectric actuators are analyzed by Vel and Batra (2000).A new theory is proposed by Pagano (1978) to define the complete stress field within an arbitrary composite laminates. The theory is based upon an extension of Reissners variational principle to laminated bodies. Kant and Swaminathan (2000) studied estimation of transverse/interlaminar stresses in laminated composites-A selected review and survey of current developments. Barbero and Reddy (1990) investigated an accurate determination of stresses in thick laminates using generalized plate theory. Knight and Qi (1997) gives restatement of first-order shear deformation theory for laminated plates. Kant and Shiyekar (2008) presented cylindrical flexure of piezoelectric plates by solving second order ordinary differential equation satisfying electric boundary conditions along thickness direction of piezoelectric layer. Shu and Soldatos(2000) given applicability of a new stress analysis method towards the accurate determination of the detailed stress distributions in angle-ply laminated plates subjected to cylindrical bending subjected to different sets of edge boundary conditions. Aydogdu (2009) derived a new shear deformation theory for 3D elasticity problems in laminated composite plates. Baillargeon and Vel (2005) derived an exact three-dimensional solution is obtained for the cylindrical bending. vibration of simply supported laminated composite plates with an embedded piezoelectric shear actuator. Chen and Lee (2004) investigated the bending and free vibration of simply supported angle-ply piezoelectric laminates in cylindrical bending. Pan and Heyliger (2002) given analytical solutions for the cylindrical bending of multilayered, linear, and anisotropic magneto-electroelastic plates under simple-supported edge conditions. Exact solution for composite laminates in cylindrical bending is to be presented by Pagano (1978).An elastic analysis of laminated composite plates forced into cylindrical bending by the application of voltages to piezoelectric actuators attached to the top and bottom surfaces is performed using the equation of linear elasticity derived by Zhou and Tiersten (1994).New theory for laminated composites applied by Bert (1984). Shear deformation for heterogeneous anisotropic plates studied by Whitney and Pagano (1970). Exact solution for cylindrical bending of laminated plates with piezoelectric actuators studied by Vel and Batra (2001).

2. Theoretical formulations

In design problems of rectangular plates, it is necessary to ensure that the plate will withstand the applied static loads by developing stresses and deflection which are well within the prescribed limits. Cauchy generalized Hooke's law to three dimensional elastic bodies and stated that the 6-components of stress are linearly related to the 6-components of strain. The stress-strain relationship written in matrix form, where the 6-components of stress and strain are organized into column vectors,

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{31} & C_{32} & C_{34} & C_{35} & C_{36} & C_{37}\\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46}\\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56}\\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}\quad \{\sigma\}=[C]\,x\,\{\varepsilon\} \tag{2}$$

where C is the stiffness matrix, 'S' is the compliance matrix, and $S = C^{-1}$.

2.1 Isotropic material

Such materials have only 2 independent variables (i.e. elastic constants) in their stiffness and compliance matrices, as opposed to the 21 elastic constants in the general anisotropic case.

The two elastic constants are usually expressed as the Young's modulus E and the Poisson's ratio ν (or 'm'). However, the alternative elastic constants bulk modulus (*K*) and/or shear modulus (*G*) can also be used. For isotropic materials, *G* and *K* can be found from *E* and ν by a set of equations, and vice-versa.

Hooke's law for isotropic materials in compliance matrix form is given by,

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ -\nu & 1 & -\nu & 0 & 0 & 0\\ -\nu & -\nu & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1+\nu & 0 & 0\\ 0 & 0 & 0 & 0 & 1+\nu & 0\\ 0 & 0 & 0 & 0 & 0 & 1+\nu\end{bmatrix}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix} \tag{3}$$

2.2 Orthotropic materials

Such materials have only 2 independent variables (i.e. elastic constants) in their stiffness and compliance matrices, as opposed to the 21 elastic constants in the general anisotropic case. The two elastic constants are usually expressed as the Young's modulus E and the Poisson's ratio ν (or 'm').However, the alternative elastic constants bulk modulus (*K*) and/or shear modulus (*G*) can also be used. For isotropic materials, *G* and *K* can be found from *E* and ν by a set of equations, and vice-versa. The 9-elastic constants in orthotropic constitutive equations are comprised of 3- Young's moduli E_x, E_y, E_z the 3-Poisson's ratios n_{xy}, n_{yz}, n_{xz} and the 3-shear moduli G_{xy}, G_{yz}, G_{xz} .The compliance matrix takes the form,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{xz}} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} \quad \text{where } \frac{\nu_{yz}}{E_y} = \frac{\nu_{zy}}{E_z}, \frac{\nu_{zx}}{E_z} = \frac{\nu_{xz}}{E_x}, \frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y} \tag{4}$$

2.3 Problem formulation:

A complete analytical formulation and solution for a laminate under cylindrical bending simply (diaphragm) supported along 'x' axis is presented. The geometry of the laminate under cylindrical bending is such that the side 'a' is along 'x' axis and side 'b' is on 'y' axis, which is assumed to be infinite. The thickness of the laminate under cylindrical bending is denoted by 'h' and is coinciding on 'z' axis. The reference mid-plane of the laminate under cylindrical bending is at $h/2$ from top or bottom surface of the laminate as shown in the Figure 3.1. The formulation is assuming fiber direction *1* of the lamina is coinciding with 'x' axis of the laminate under cylindrical bending. Figure also illustrates the mid-plane positive set of displacements along x-y-$z$axes.In laminate under cylindrical bending, the dimension (along y direction) is considered as infinite compared to other dimensions (along x and z directions). In such problems, the strains along y direction are very small as compared to x and z directions and can be neglected. Then problem is assumed to be in two-dimensional and in a state of plane strain. Neglecting the strains along y direction i.e. $\varepsilon_y \approx 0; \gamma_{xy} \approx 0; \gamma_{xz} \approx 0$, the stress-strain relationship for a two-dimensional orthotropic body under plane strain condition can be stated as $\varepsilon_y = 0; \gamma_{xy} = 0; \gamma_{xz} = 0$

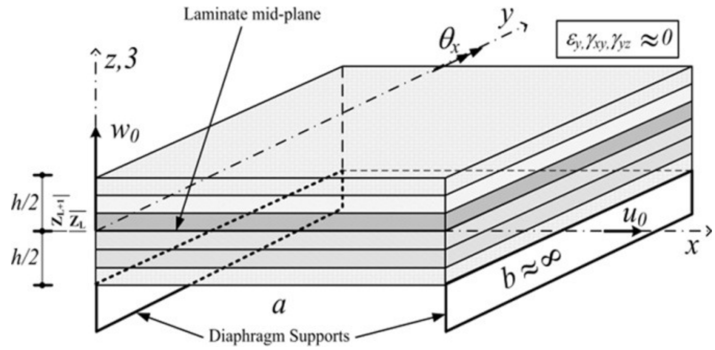

Figure 1.Geometry of a laminate under cylindrical bending with positive set of displacements and axes.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\ Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ 0 \\ \varepsilon_z \\ 0 \\ 0 \\ \gamma_{xz} \end{Bmatrix} \tag{5}$$

From above equation it also concludes that $\tau_{xy} = 0$ and $\tau_{yz} = 0$

Rearranging the equation in a matrix form, it becomes

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{13} & 0 \\ \bar{Q}_{13} & \bar{Q}_{33} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \end{Bmatrix} \text{ where, } \begin{matrix} \bar{Q}_{11} = Q_{11} = C_{11}c^4 + 2(C_{12} + 2C_{44})s^2c^2 + C_{22}s^4; \\ \bar{Q}_{13} = Q_{13} = C_{13}c^2 + C_{23}s^2; \\ \bar{Q}_{33} = Q_{33} = C_{33}; \bar{Q}_{66} = Q_{66} = C_{55}s^2 + C_{66}c^2 \end{matrix} \tag{6}$$

$$C_{11} = \frac{E_1(1-\nu_{23}\nu_{32})}{\Delta}; C_{13} = \frac{E_1(\nu_{31}+\nu_{21}\nu_{32})}{\Delta}; C_{33} = \frac{E_3(1-\nu_{12}\nu_{21})}{\Delta}; C_{66} = G_{13}.; \Delta = (1-\nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{12}\nu_{23}\nu_{31})$$

2.4 Displacement model

FOST model is also formulated and the displacement model is in the following form.

ModelFOST:

$$u(x,z) = u_0(x) + z\theta_x(x)$$
$$w(x,z) = w_0(x) \tag{7}$$

The parameter u_0 is in-plane displacement and w_0 is the transverse displacement on the middle plane. $\theta_x(x)$ is the rotation of the normal to the middle-plane about *y-axis*.

2.5 Governing equations of equilibrium

Using the principle of minimum potential energy derived the equation of equilibrium. In analytical form it can be written as, $\delta(U+V) = 0$ where U is the total strain energy due to deformation, V is the potential of the external loads and $U+V= \pi$ is the total potential energy and δ is the variational symbol. Substituting the appropriate energy expressions in the above equation, the final expression can be written as,

$$\left[\int_{-h/2}^{+h/2} \int_l (\sigma_x \delta\varepsilon_x + \sigma_z \delta\varepsilon_z + \tau_{xz}\delta\gamma_{xz}) dx dz - \int_l q_0^+ \delta w^+ dx \right] = 0 \tag{8}$$

Where, $w^+ = w_0$ is the transverse displacement at top surface of the plate. q_0^+ is the transverse load applied at top of the plate. Integrating the above equation by parts and collecting the coefficients of $\delta u_0, \delta w_0, \delta\theta_x$, the following equations of equilibrium are obtained.

$$\delta u_0 : \frac{\partial N_x}{\partial x} = 0, \delta w_0 : \frac{\partial Q_x}{\partial x} + q_0^+ = 0, \delta\theta_x : \frac{\partial M_x}{\partial x} - Q_x = 0 \tag{9}$$

The stress resultants in terms are defined $M_x = \sum_{l=1}^{n} \int_{Z_L}^{Z_{L+1}} \sigma_x z dz, Q_x = \sum_{l=1}^{n} \int_{Z_L}^{Z_{L+1}} \tau_{xz} dz, N_x = \sum_{l=1}^{n} \int_{Z_L}^{Z_{L+1}} \sigma_x dz,$. (10)

2.6 Analytical solution for plane strain condition

Following are the boundary conditions used for two opposite infinite simply (diaphragm) supported edges: At edges $x = 0$ and $x = a$:

$w_0= 0$, $M_x= 0$, $N_x= 0$,

Navier's solution procedure is adopted to evaluate displacement variables. Displacements, which satisfy the above boundary conditions, can be assumed as follows:

$$u_0 = \sum_{m=1,3,5}^{\infty} u_{0_m} \cos(\frac{m\pi x}{a}),\ w_0 = \sum_{m=1,3,5}^{\infty} w_{0_m} \sin(\frac{m\pi x}{a}),\ \theta_x = \sum_{m=1,3,5}^{\infty} \theta_{x_m} \cos(\frac{m\pi x}{a}) \tag{11}$$

Strains are evaluated from strain displacement relationship and these are

$$\varepsilon_x = \frac{-\pi(u_{0m} + z(\theta_{xm}))}{a}\sin(\frac{\pi x}{a});\quad \gamma_{xz} = \frac{(a(\theta_{xm}) + \pi(w_{0m})}{a}\cos(\frac{\pi x}{a}) \tag{12}$$

From constitutive relationship of plane strain condition (Equation), stresses are obtained as

$$\sigma_x = -\frac{\pi Q_{11}\left(u_{0_{mn}} + z\left(\theta_{x_{mn}}\right)\right)\sin\left(\frac{\pi x}{a}\right)}{a},\quad \tau_{xz} = \frac{\bar{Q}_{66}\left(a\left(\theta_{x_{mn}}\right) + \left(w_{0_{mn}}\right)\right)\cos\left(\frac{\pi x}{a}\right)}{a} \tag{13}$$

Stress resultants are obtained as

$$M_x = -\frac{1}{240a}\left(h^2\left(h\pi\bar{Q}_{11}\left(20\theta_{x_{mn}} + 3h^2\theta_{x_{mn}}\right)\right)\sin\left(\frac{\pi x}{a}\right) \tag{14}$$

$$Q_x = \frac{h\bar{Q}_{66}\left(12\pi w_{0_{mn}} + 12a\theta_{x_{mn}}\right)\cos\left(\frac{\pi x}{a}\right)}{12a},\qquad N_x = \frac{h\left(-\pi\bar{Q}_{11}\left(12u_{0_{mn}}\right)\right)\sin\left(\frac{\pi x}{a}\right)}{12a}$$

The intensity of transverse loading can be expressed in the form of Fourier series as,

$$p(x) = \sum_m P_{0m}\sin\frac{m\pi x}{a} \tag{15}$$

Where p_{0m} is the peak intensity of distributed loading corresponding to m[th]harmonic

All the numerical results presented for this example are normalized as per the following.

$$\bar{u}\left(0,\pm\frac{z}{H}\right) = \frac{100E_2}{q_0S^4H}(u);\quad \bar{w}(a/2,0) = \frac{100E_2}{q_0S^4H}(w);\quad \bar{\sigma}_x\left(0,\pm\frac{z}{H}\right) = \frac{q_0}{S^2}(\sigma_x);$$

$$\bar{\sigma}_z\left(a/2,\pm\frac{z}{H}\right) = \frac{(\sigma_z)}{q_0};\quad \bar{\tau}_{xz}\left(0,\pm\frac{z}{H}\right) = \frac{(\tau_{xz})}{q_0}. \tag{16}$$

3. Numerical investigations

Table 1. Boundary conditions (BCs)

Edg	BCs on displacement field	BCs on stress field
x = 0	w=0	σx = 0
x = a/2	u = 0	τ_{xz}= 0
z = h/2	-	σz = p(x) ; τ_{xz}=0
z = -h/2	-	σz = 0 ; τ_{xz}= 0

Table 2. Material properties for laminated composites.

Source	Material properties	
Pagano (1969)	E_1 = 172.4 GPa	ν_{12} = 0.25
	G_{12} = 3.45 GPa	
	E_2 = 6.89 GPa	ν_{13} = 0.25
	G_{13} = 3.45 GPa	
	E_3 = 6.89 GPa	ν_{23} = 0.25
	G_{23} = 1.378 GPa	

Numerical investigation has been done for layered plates with orthotropic layers simply (diaphragm) supported on two edges at *x* = 0 and *x* = *a*, with material properties as shown in Table 2. The different configurations of the plates are,

1. Single layer of homogeneous isotropic plate.
2. Single layer of homogeneous orthotropic unidirectional (0^0) plate.

3.

3.1 Single layer of homogeneous isotropic plate.

Table 3. Normalized transverse displacement (w), inplane normal stress (σ_x) and transverse shear stress (τ_{xz}) of an isotropic plate under cylindrical bending

Aspect ratio	Source	σ_x (a/2 , h/2)	σ_x (a/2 , -h/2)	τ_{xz} (max)	W (a/2 , 0)
4	Kant[1] (2008)	0.6223	-0.6192	0.4750	12.947
	Pagano(1969)	0.6223	-0.6192	0.4750	12.947
	Kant[2] (2008)	0.7600	-0.7625	0.7035	16.364
	Present analysis	0.6321	-0.6321	0.4753	13.593
10	Kant[1] (2008)	0.6100	-0.6100	0.4771	11.489
	Pagano(1969)	0.6100	-0.6100	0.4771	11.490
	Kant[2] (2008)	0.7516	-0.7519	0.7243	14.563
	Present analysis	0.6162	-0.6162	0.4873	12.530
20	Kant[1] (2008)	0.6048	-0.6084	0.4774	11.279
	Pagano(1969)	0.6084	-0.6084	0.4774	11.280
	Kant[2] (2008)	0.7503	-0.7504	0.7274	14.303
	Present analysis	0.6191	-0.6191	0.4729	12.378
30	Kant[1] (2008)	0.6081	-0.6081	0.4774	11.241
	Pagano(1969)	0.6081	-0.6081	0.4774	11.241
	Kant[2] (2008)	0.7501	-0.7501	0.7279	14.255
	Present analysis	0.6093	-0.6093	0.4700	12.350
40	Kant[1] (2008)	0.6081	-0.6081	0.4774	11.227
	Pagano(1969)	0.9081	-0.6081	0.4774	11.228
	Kant[2] (2008)	0.7500	-0.7500	0.7281	14.239
	Present analysis	0.6029	-0.6129	0.4920	12.340
50	Kant[1] (2008)	0.6080	-0.6080	0.4774	11.220
	Pagano(1969)	0.6080	-0.6080	0.4774	11.220
	Kant[2] (2008)	0.7500	-0.7500	0.7282	14.232
	Present analysis	0.7120	-0.7120	0.7001	12.336

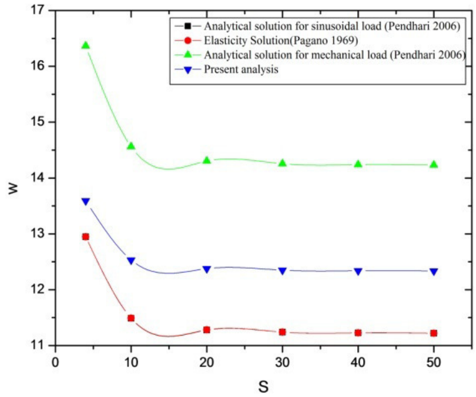

Figure 2 Comparison of normalized variation of transverse displacement (w)for various span thickness ratios (S) for an isotropic plate under cylindrical bending.

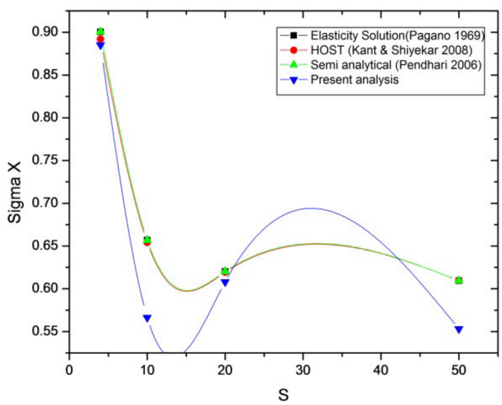

Figure 3 Comparison of normalized variation of inplane stress (σ_x) for various span thickness ratios (S) for an isotropic plate under cylindrical bending

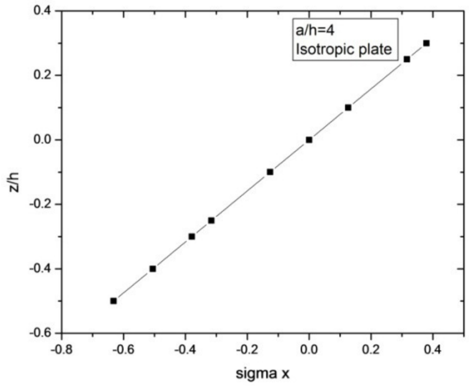

Figure 4 Variation of normalized inplane normal stress (σ_x) through the thickness of an isotropic plate under cylindrical bending.

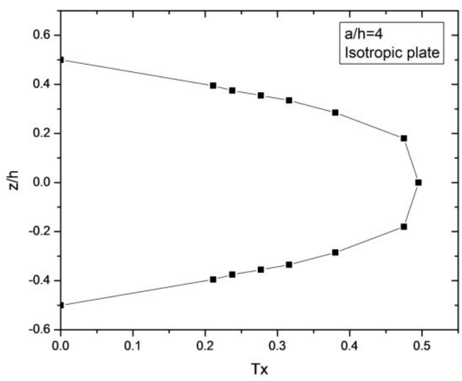

Figure 5 Variation of normalized transverse shear stress ($\boldsymbol{\tau_x}$) through the thickness of an isotropic plate under cylindrical bending

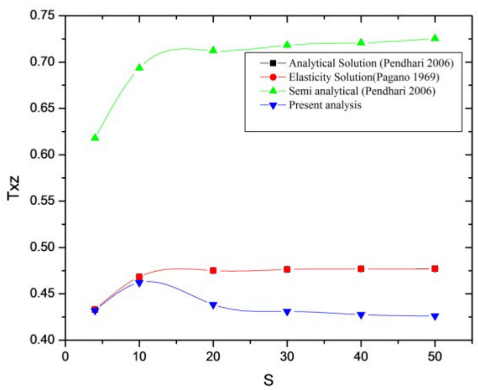

Figure 6 Comparison of normalized variation of transverse shear stress (w) for various span thickness ratios (S) for an isotropic plate under cylindrical bending.

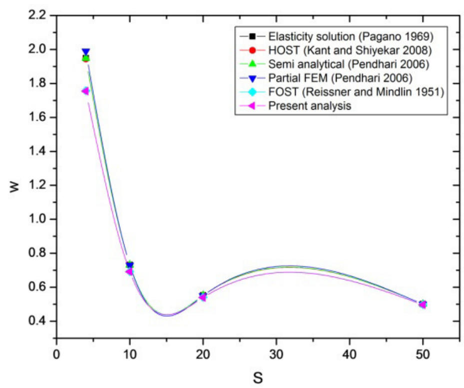

Figure 7. Comparison of normalized variation of transverse displacement (w) for various span thickness ratios (S) for an orthotropic plate under cylindrical bending

Table 4. Normalized transverse displacement (w), inplane normal stress (σ_x) and transverse shear stress (τ_{xz}) of an orthotropic plate under cylindrical bending

Aspect ratio	Source	σ_x (a/2 , h/2)	σ_x (a/2 , -h/2)	τ_{xz} (max)	w (a/2 , 0)
4	Pagano (1969)	0.9006	-0.8481	0.4330	1.9490
	Kant & Shiyekar (2008)	0.8920	-0.8497	0.4258	1.9443
	Pendhari – Semiana.(2006)	0.9006	-0.8481	0.4328	1.9489
	Pendhari – FEM (2006)	0.8204	-0.7710	0.4759	1.9906
	Reissner&Mindlin (1951)	0.6079	-0.6079	0.4774	1.756
	Present analysis	0.8851	-0.8851	0.4321	1.7542
10	Pagano(1969)	0.6569	-0.6551	0.4683	0.7319
	Kant &Shiyekar (2008)	0.6541	-0.6587	0.4679	0.7311
	Pendhari – Semiana.(2006)	0.6569	-0.6551	0.4683	0.7319
	Pendhari – FEM (2006)	0.6432	-0.6414	0.4788	0.7306
	Reissner & Mindlin (1951)	0.6079	-0.6079	0.4774	0.6921
	Present analysis	0.5665	-0.5665	0.4621	0.6912

20	Pagano(1969)	0.6203	-0.6202	0.4751	0.5519
	Kant &Shiyekar (2008)	0.6194	-0.6212	0.4750	0.5514
	Pendhari – Semiana.(2006)	0.6203	-0.6202	0.4751	0.5519
	Pendhari – FEM (2006)	0.6070	-0.6068	0.4820	0.5499
	Reissner & Mindlin (1951)	0.6079	-0.6079	0.4774	0.5401
	Present analysis	0.7080	-0.7080	0.4385	0.5393
50	Pagano (1969)	0.6095	-0.6095	0.4769	0.5012
	Kant &Shiyekar (2008)	0.6097	-0.6100	0.4770	0.5008
	Pendhari – Semiana.(2006)	0.6095	-0.6095	0.4769	0.5012
	Pendhari – FEM (2006)	0.6000	-0.6000	0.4847	0.5000
	Reissner & Mindlin (1951)	0.6079	-0.6079	0.4774	0.4975
	Present analysis	0.6060	-.6060	0.4262	0.4967

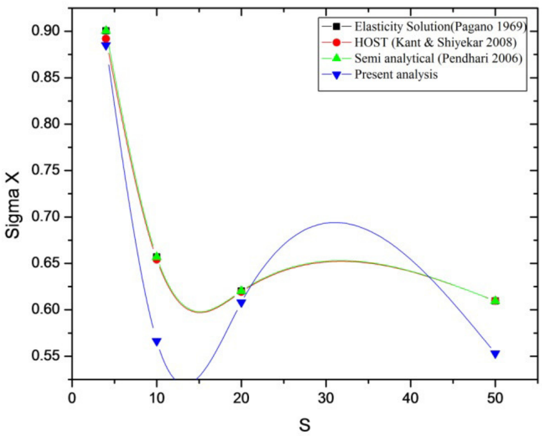

Figure 8. Comparison of normalized variation of inplane stress (σ_x) for various span thickness ratios (S) for an orthotropic plate under cylindrical bending.

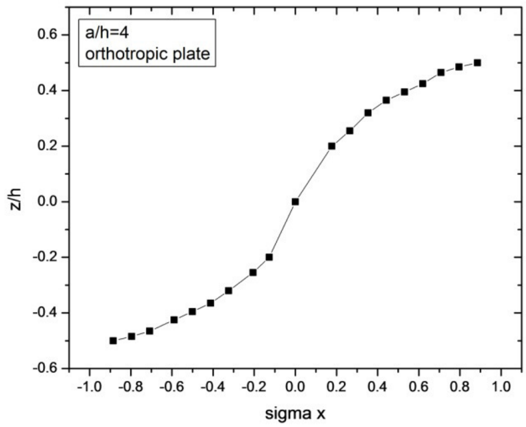

Figure 9. Comparison of normalized variation of inplane stress (σ_x) for various span thickness ratios (S) for an orthotropic plate under cylindrical bending.

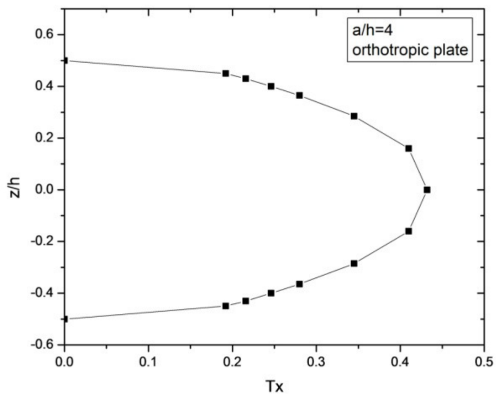

Figure 10. Variation of normalized transverse shear stress ($\boldsymbol{\tau_{xz}}$) through the thickness of an orthotropic plate under cylindrical bending.

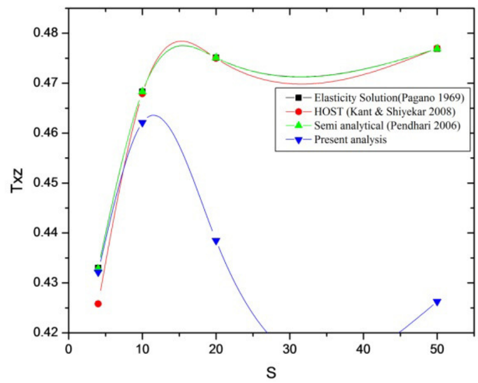

Figure 11. Comparison of normalized variation of transverse shear stress ($\boldsymbol{\tau_{xz}}$) for various span thickness ratios (S) for an orthotropic plate under cylindrical bending.

4. Conclusions

Composite plate is being analyzed by using 2D First order shear deformation theory. Laminates subjected to transversely distributed load under cylindrical bending has been presented in this report. The formulation is simplified both transverse stresses and displacements is enforced with thickness of the laminate. The solution observed gives excellent results with the elasticity solution. Since loading term is expanded in the form of Fourier series, any system of loading can be handled with this formulation. The present results are compared with exact solution and other in given literature. The results obtained from all theories are approximately same and it changes with respect to aspect ratio.

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